Identidades

Demuestra las siguientes identidades:

1. $(sen\alpha + cos\alpha)^2 - (sen\alpha - cos\alpha)^2 = 4sen\alpha cos\alpha$

2. $sen\alpha \cdot cos^2\alpha + sen^3\alpha = sen\alpha$

3. $\dfrac{sen\alpha}{1+cos\alpha} + \dfrac{sen\alpha}{1-cos\alpha} = \dfrac{2}{sen\alpha}$

4. $\dfrac{\cos(\alpha+\beta)+\cos(\alpha-\beta)}{sen(\alpha+\beta)+sen(\alpha-\beta)} = \dfrac{1}{tg\alpha}$

5. $cos3\alpha = 4cos^3\alpha - 3cos\alpha$

6. $cos\alpha \cos(\alpha-\beta) + sen\alpha sen(\alpha-\beta) = cos\beta$

7. $\dfrac{cos\alpha+sen\alpha}{cos\alpha-sen\alpha} \cdot cos2\alpha = 1 + sen2\alpha$

$Sol: 1)(sen\alpha + cos\alpha)^2 - (sen\alpha - cos\alpha)^2 = sen^2\alpha + 2sen\alpha cos\alpha + cos^2\alpha - (sen^2\alpha - 2sen\alpha cos\alpha + cos^2\alpha)$
$= sen^2\alpha + 2sen\alpha cos\alpha + cos^2\alpha - sen^2\alpha + 2sen\alpha cos\alpha - cos^2\alpha = 4sen\alpha cos\alpha$

$2) sen\alpha \cdot cos^2\alpha + sen^3\alpha = sen\alpha(cos^2\alpha + sen^2\alpha) = sen\alpha \cdot 1 = sen\alpha$

$3) \dfrac{sen\alpha}{1+cos\alpha} + \dfrac{sen\alpha}{1-cos\alpha} = \dfrac{sen\alpha - sen\alpha cos\alpha + sen\alpha + sen\alpha cos\alpha}{(1+cos\alpha)(1-cos\alpha)} = \dfrac{2sen\alpha}{1-cos^2\alpha} = \dfrac{2sen\alpha}{sen^2\alpha} = \dfrac{2}{sen\alpha}$

$4)\dfrac{\cos(\alpha+\beta)+\cos(\alpha-\beta)}{sen(\alpha+\beta)+sen(\alpha-\beta)} = \dfrac{cos\alpha cos\beta - sen\alpha sen\beta + cos\alpha cos\beta + sen\alpha sen\beta}{sen\alpha cos\beta + cos\alpha sen\beta + sen\alpha cos\beta - cos\alpha sen\beta} = \dfrac{2cos\alpha cos\beta}{2sen\alpha cos\beta} = \dfrac{cos\alpha}{sen\alpha} = \dfrac{1}{tg\alpha}$

$5) cos3\alpha = \cos(2\alpha + \alpha) = cos2\alpha cos\alpha - sen2\alpha sen\alpha = (cos^2\alpha - sen^2\alpha)cos\alpha - 2sen\alpha cos\alpha sen\alpha = cos^3\alpha - sen^2\alpha cos\alpha - 2sen^2\alpha cos\alpha = cos^3\alpha - 3sen^2\alpha cos\alpha = cos^3\alpha - 3(1 - cos^2\alpha)cos\alpha = cos^3\alpha - 3cos\alpha + 3cos^3\alpha = 4cos^3\alpha - 3cos\alpha$

$6) cos\alpha \cos(\alpha-\beta) + sen\alpha sen(\alpha-\beta) = cos\alpha(cos\alpha cos\beta + sen\alpha sen\beta) + sen\alpha(sen\alpha cos\beta - cos\alpha sen\beta)$
$= cos^2\alpha cos\beta + sen\alpha sen\beta cos\alpha + sen^2\alpha cos\beta - sen\alpha cos\beta cos\alpha = cos\beta(cos^2\alpha + sen^2) = cos\beta$

$7) \dfrac{cos\alpha+sen\alpha}{cos\alpha-sen\alpha} \cdot cos2\alpha = \dfrac{cos\alpha+sen\alpha}{cos\alpha-sen\alpha}(cos^2\alpha - sen^2\alpha) = \dfrac{cos\alpha+sen\alpha}{cos\alpha-sen\alpha}(cos\alpha+sen\alpha)(cos\alpha-sen\alpha)$
$= (cos\alpha+sen\alpha)(cos\alpha+sen\alpha) = cos^2\alpha + 2cos\alpha sen\alpha + sen^2\alpha = 1 + 2sen\alpha cos\alpha$
$= 1 + sen2\alpha$

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